Introduction to Description Logic
and Query Rewriting

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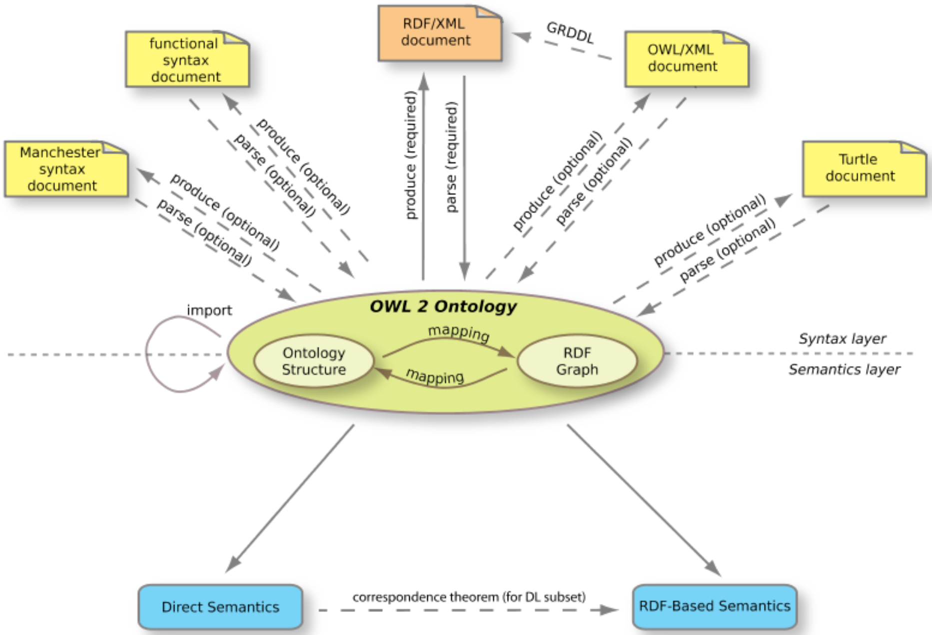
The Web Ontology Language



Part 1

DL Basics

Description Logics by Example

SubClassOf(**ObjectIntersectionOf**(Person,
ObjectSomeValuesFrom(takesCourse, Course)), Student)

$$\text{Person} \sqcap \exists \text{takesCourse}.\text{Course} \sqsubseteq \text{Student}$$

SubObjectPropertyOf(mastersDegreeFrom, degreeFrom)

$$\text{mastersDegreeFrom} \sqsubseteq \text{degreeFrom}$$

SubClassOf(**ObjectSomeValuesFrom**(
ObjectInverseOf(takesCourse), **owl:Thing**), Course)

$$\text{takesCourse}^{-}.\top \sqsubseteq \text{Course}$$

ClassAssertion(Student, john)

$$\text{Student}(\text{john})$$

ObjectPropertyAssertion(takesCourse, john, sw)

$$\text{takesCourse}(\text{john}, \text{sw})$$

DL Syntax

concepts (classes)

$$C \quad ::= \quad \underbrace{A_i}_{\text{concept name}} \quad | \quad \underbrace{\top}_{\text{owl:Thing}} \quad | \quad \underbrace{\bot}_{\text{owl:Nothing}} \quad |$$

$$\underbrace{\neg C}_{\text{ObjectComplementOf}(C)} \quad | \quad \underbrace{C_1 \sqcap C_2}_{\text{ObjectIntersectionOf}(C_1,C_2)} \quad | \quad \underbrace{C_1 \sqcup C_2}_{\text{ObjectUnionOf}(C_1,C_2)} \quad |$$

$$\underbrace{\exists R.C}_{\text{ObjectSomeValuesFrom}(R,C)} \quad | \quad \underbrace{\forall R.C}_{\text{ObjectAllValuesFrom}(R,C)}$$

roles (object properties)

$$R \quad ::= \quad \underbrace{P_i}_{\text{role name}} \quad | \; P_i^-$$

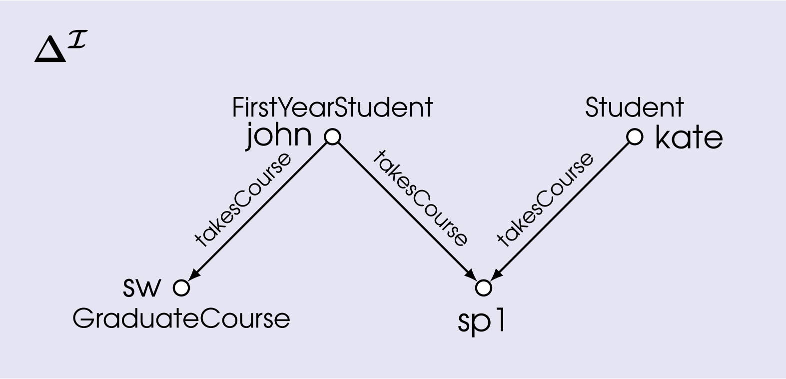
TBox $\mathcal{T}$ $\quad \underbrace{C_1 \sqsubseteq C_2}_{\text{SubClassOf}(C_1,C_2)}$ and $\underbrace{R_1 \sqsubseteq R_2}_{\text{SubObjectPropertyOf}(R_1,R_2)}$

ABox $\mathcal{A}$ $\quad C(a)$ and $R(a, b)$

$\boxed{\textbf{knowledge base } \mathcal{K} = (\mathcal{T}, \mathcal{A}) \qquad \text{(ontology)}}$

DL Semantics

interpretation $\mathcal{I} = (\underbrace{\Delta^{\mathcal{I}}}_{\text{domain}}, \cdot^{\mathcal{I}})$



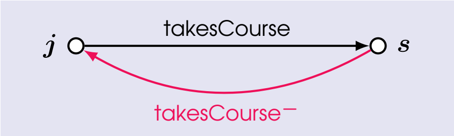
$\cdot^{\mathcal{I}}$ (interpretation function)

individuals a_i $\longrightarrow$ elements $a_i^{\mathcal{I}} \in \Delta^{\mathcal{I}}$

concept names A_i $\longrightarrow$ sets $A_i^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$

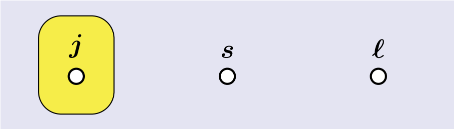
role names P_i $\longrightarrow$ binary relations $P_i^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$

DL Semantics (2)



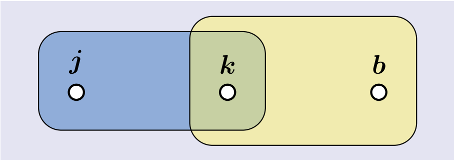
$$(P^-)^\mathcal{I} \;=\; \{(v,u) \mid (u,v) \in P^\mathcal{I}\}$$

$\top^\mathcal{I} \;=\; \Delta^\mathcal{I}$ and $\bot^\mathcal{I} \;=\; \emptyset$



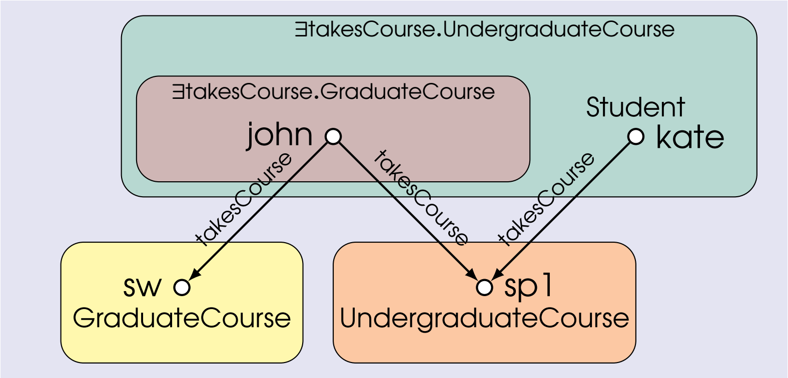
$$(\neg C)^\mathcal{I} \;=\; \Delta^\mathcal{I} \setminus C^\mathcal{I}$$

$(C_1 \sqcap C_2)^\mathcal{I} \;=\; C_1^\mathcal{I} \cap C_2^\mathcal{I}$ and $(C_1 \sqcup C_2)^\mathcal{I} \;=\; C_1^\mathcal{I} \cup C_2^\mathcal{I}$



DL Semantics (3)

$(\exists R.C)^{\mathcal{I}} \;=\; \{\, u \mid$ there is $v \in C^{\mathcal{I}}$ such that $(u, v) \in R^{\mathcal{I}} \,\}$



$(\forall R.C)^{\mathcal{I}} \;=\; \{\, u \mid v \in C^{\mathcal{I}},$ for all v with $(u, v) \in R^{\mathcal{I}} \,\}$

$\forall R.C = \neg\exists R.\neg C$

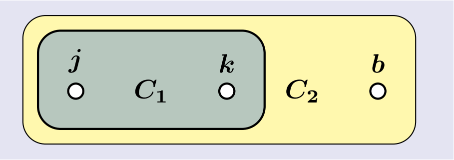
NB. "for all" is true when there are no v with $(u, v) \in R^{\mathcal{I}}$

e.g., sp1 $\in$ ($\forall$takesCourse.UndergraduateCourse)$^{\mathcal{I}}$

sp1 $\in$ ($\forall$takesCourse.$\bot$)$^{\mathcal{I}}$

DL Semantics (4)

$$\mathcal{I} \models C_1 \sqsubseteq C_2 \quad \iff \quad C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}}$$



$$\mathcal{I} \models R_1 \sqsubseteq R_2 \quad \iff \quad R_1^{\mathcal{I}} \subseteq R_2^{\mathcal{I}}$$

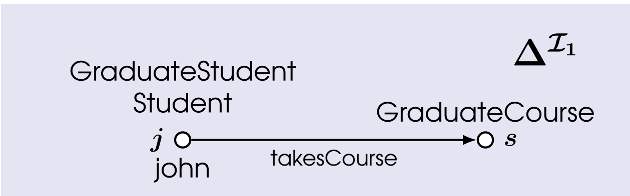
$$\mathcal{I} \models C(a) \quad \iff \quad a^{\mathcal{I}} \in C^{\mathcal{I}}$$

$$\mathcal{I} \models R(a, b) \quad \iff \quad (a^{\mathcal{I}}, b^{\mathcal{I}}) \in R^{\mathcal{I}}$$

$\mathcal{I}$ is a **model** of $(\mathcal{T}, \mathcal{A})$ if $\mathcal{I} \models \alpha$, for all inclusions α in $\mathcal{T}$ and assertions α in $\mathcal{A}$

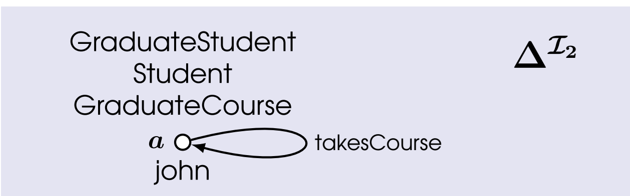
Open World Assumption

$$\begin{aligned}\mathcal{T} &= \{\ \text{GraduateStudent} \sqsubseteq \text{Student}\\ &\qquad \text{GraduateStudent} \sqsubseteq \exists \text{takesCourse}.\text{GraduateCourse}\ \}\\ \mathcal{A} &= \{\ \text{GraduateStudent(john)}\ \}\end{aligned}$$



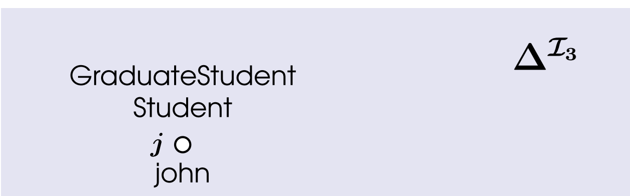
$\text{john}^{\mathcal{I}_1} = j$
$\text{GraduateStudent}^{\mathcal{I}_1} = \{j\}$
$\text{Student}^{\mathcal{I}_1} = \{j\}$
$\text{GraduateCourse}^{\mathcal{I}_1} = \{s\}$
$\text{takesCourse}^{\mathcal{I}_1} = \{(j, s)\}$

is a **model**



$\text{john}^{\mathcal{I}_2} = a$
$\text{GraduateStudent}^{\mathcal{I}_2} = \{a\}$
$\text{Student}^{\mathcal{I}_2} = \{a\}$
$\text{GraduateCourse}^{\mathcal{I}_2} = \{a\}$
$\text{takesCourse}^{\mathcal{I}_2} = \{(a, a)\}$

is a **model**



$\text{john}^{\mathcal{I}_3} = j$
$\text{GraduateStudent}^{\mathcal{I}_3} = \{j\}$
$\text{Student}^{\mathcal{I}_3} = \{j\}$
$\text{GraduateCourse}^{\mathcal{I}_3} = \emptyset$
$\text{takesCourse}^{\mathcal{I}_3} = \emptyset$

is **not a model**

Reasoning: Consistency

a knowledge base $\mathcal{K}$ is **satisfiable** (or **consistent**)

if there exists <u>at least one</u> model of $\mathcal{K}$

(in other words, $\mathcal{K}$ implies **no contradictions**)

Example

$\mathcal{T}$:

$\mathsf{UndergraduateStudent} \sqsubseteq \forall \mathsf{takesCourse}.\mathsf{UndergraduateCourse}$

$\mathsf{UndergraduateCourse} \sqcap \mathsf{GraduateCourse} \sqsubseteq \bot$

$\mathcal{A}$:

$\mathsf{UndergraduateStudent}(\mathsf{john})$

$\mathsf{takesCourse}(\mathsf{john}, \mathsf{sw})$

$\mathsf{GraduateCourse}(\mathsf{sw})$

$(\mathcal{T}, \mathcal{A})$ is **inconsistent**:

John (as an undergraduate student) can take only undergraduate courses.
We know, however, that he takes a graduate course,
which cannot be an undergraduate one.

Reasoning: Entailment

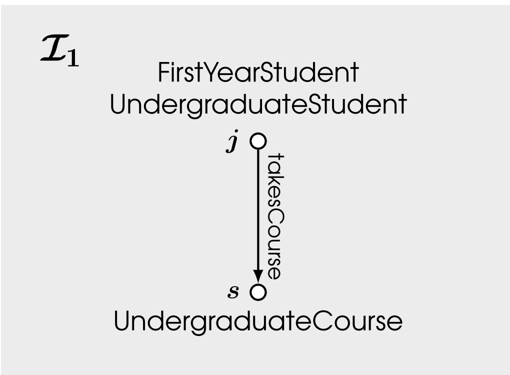
$C_1 \sqsubseteq C_2$ is **entailed by** $\mathcal{K}$ $\qquad$ $\boxed{\mathcal{K} \models C_1 \sqsubseteq C_2}$

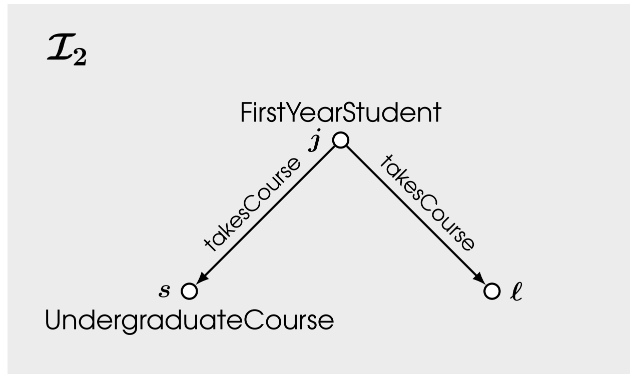
if $\mathcal{I} \models C_1 \sqsubseteq C_2$ for all models $\mathcal{I}$ of $\mathcal{K}$

(entailment for role inclusions and concept and role assertions is defined similarly)

$\mathcal{T}$: $\quad \forall$takesCourse.UndergraduateCourse $\sqsubseteq$ UndergraduateStudent

FirstYearStudent $\sqsubseteq$ $\exists$takesCourse.UndergraduateCourse.





$\mathcal{I}_1 \models \mathcal{T}$

$\mathcal{I}_1 \models$ FirstYearStudent $\sqsubseteq$ UndergraduateStudent

$\mathcal{I}_2 \models \mathcal{T}$

$\mathcal{I}_2 \models$ FirstYearStudent $\not\sqsubseteq$ UndergraduateStudent

Reasoning: Entailment (2)

C_1 is **subsumed** by C_2 with respect to $\mathcal{K}$ if $\mathcal{K} \models C_1 \sqsubseteq C_2$

Proposition $(\mathcal{T}, \mathcal{A}) \models C_1 \sqsubseteq C_2$ iff $(\mathcal{T}, \mathcal{A} \cup \{C_1(a), \neg C_2(a)\})$ is not satisfiable
for a **fresh** a

Proof iff = if and only if

($\Rightarrow$, only if) Let $(\mathcal{T}, \mathcal{A}) \models C_1 \sqsubseteq C_2$.
Assume, for the sake of contradiction, that $(\mathcal{T}, \mathcal{A} \cup \{C_1(a), \neg C_2(a)\})$ is satisfiable.
Then there is an interpretation $\mathcal{I}$ such that $\mathcal{I} \models (\mathcal{T}, \mathcal{A})$ (a model of $(\mathcal{T}, \mathcal{A})$),
$a^{\mathcal{I}} \in C_1^{\mathcal{I}}$ and $a^{\mathcal{I}} \notin C_2^{\mathcal{I}}$, contrary to $(\mathcal{T}, \mathcal{A}) \models C_1 \sqsubseteq C_2$

($\Leftarrow$, if) Let $(\mathcal{T}, \mathcal{A} \cup \{C_1(a), \neg C_2(a)\})$ be not satisfiable.
Assume, for the sake of contradiction, that $(\mathcal{T}, \mathcal{A}) \not\models C_1 \sqsubseteq C_2$.
Then there is a model of $(\mathcal{T}, \mathcal{A})$ and some $u \in C_1^{\mathcal{I}}$ such that $u \notin C_2^{\mathcal{I}}$.
Since a is fresh, we can redefine $a^{\mathcal{I}}$ by taking $a^{\mathcal{I}} = u$,
which will contradict inconsistency of $(\mathcal{T}, \mathcal{A} \cup \{C_1(a), \neg C_2(a)\})$.

a is an **instance of** C with respect to $\mathcal{K}$ if $\mathcal{K} \models C(a)$

Proposition $(\mathcal{T}, \mathcal{A}) \models C(a)$ iff $(\mathcal{T}, \mathcal{A} \cup \{\neg C(a)\})$ is not satisfiable

Conjunctive Queries (CQs)

conjunctive query

$$\boxed{q(\underbrace{\vec{x}}_{\text{answer variables}}) \quad = \quad \exists \underbrace{\vec{y}}_{\text{existentially quantified variables}} \varphi(\vec{x}, \vec{y})}$$

$\varphi(\vec{x}, \vec{y})$ is a conjunction of atoms such as $\underbrace{A}_{\text{concept name}}(z)$ and $\underbrace{P}_{\text{role name}}(z_1, z_2)$

z, z_1 and z_2 are **terms** = an individual name or a variable from $\vec{x}$ or $\vec{y}$

Example

$$q(x) = \exists y, z \; (\text{friend}(x, y) \land \text{Female}(y) \land \text{loves}(y, z) \land \text{Male}(z))$$

```
SELECT ?x
WHERE {
      ?x :friend ?y.
      ?y a :Female.
      ?y :loves ?z.
      ?z a :Male.
}
```

Certain Answers to CQs

$q(\vec{x}) = \exists \vec{y}\, \varphi(\vec{x}, \vec{y})$ is a CQ with $\vec{x} = (x_1, \dots, x_n)$

$\vec{a} = (a_1, \dots, a_n)$ is a tuple of individual names from $\mathcal{A}$

$q(\vec{a})$ is the result of replacing each x_i in $\exists \vec{y}\, \varphi(\vec{x}, \vec{y})$ with a_i

$\vec{a}$ is a **certain answer** to $q(\vec{x})$ over $(\mathcal{T}, \mathcal{A})$ $\qquad \mathcal{I} \models q(\vec{a})$

if, for any model $\mathcal{I}$ of $(\mathcal{T}, \mathcal{A})$, the sentence $q(\vec{a})$ is true in $\mathcal{I}$

q without answer variables is **Boolean**

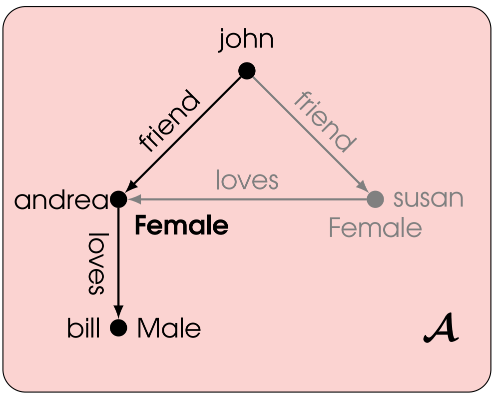
a certain answer to q over $(\mathcal{T}, \mathcal{A})$ is 'yes' if $(\mathcal{T}, \mathcal{A}) \models q$ and 'no' otherwise

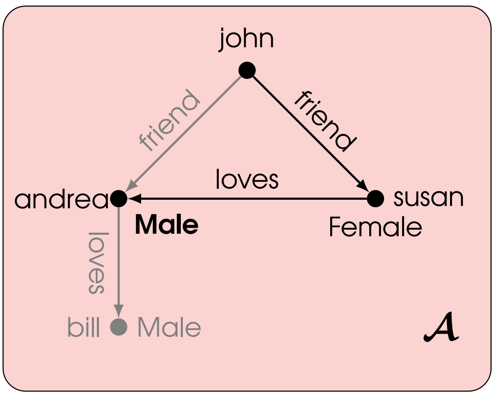
Andrea's Example

$\mathcal{T}$: $\top \sqsubseteq \mathsf{Male} \sqcup \mathsf{Female}, \quad \mathsf{Male} \sqcap \mathsf{Female} \sqsubseteq \bot$

$\mathcal{A}$: friend(john, susan), friend(john, andrea), Female(susan)
loves(susan, andrea), loves(andrea, bill), Male(bill)

$$q(x) = \exists y, z\ (\mathsf{friend}(x, y) \wedge \mathsf{Female}(y) \wedge \mathsf{loves}(y, z) \wedge \mathsf{Male}(z))$$





NB: the same as finding instances of $\exists$friend.(Female $\sqcap$ $\exists$loves.Male)

From OWL to DL

ObjectPropertyDomain(takesCourse, Student)

$$\exists \text{takesCourse}.\top \sqsubseteq \text{Student}$$

ObjectPropertyRange(takesCourse, Course)

$$\exists \text{takesCourse}^{-}.\top \sqsubseteq \text{Course}$$

or

$$\top \sqsubseteq \forall \text{takesCourse}.\text{Course}$$

Proposition The following are equivalent (have the same models)

$C_1 \sqsubseteq \forall P.C_2$	and	$\exists P^{-}.C_1 \sqsubseteq C_2$
$C_1 \sqcup C_2 \sqsubseteq C$	and	$\{C_1 \sqsubseteq C, \ C_2 \sqsubseteq C\}$
$C \sqsubseteq C_1 \sqcap C_2$	and	$\{C \sqsubseteq C_1, \ C \sqsubseteq C_2\}$
$C_1 \sqsubseteq \neg C_2$	and	$C_1 \sqcap C_2 \sqsubseteq \bot$

From OWL to DL (2)

EquivalentClasses$(C_1, C_2, \dots, C_n)$

$$C_1 \sqsubseteq C_2, \quad C_2 \sqsubseteq C_3, \quad \dots, \quad C_{n-1} \sqsubseteq C_n, \quad C_n \sqsubseteq C_1$$

NB: $A \equiv B$ stands for '$A \sqsubseteq B$ and $B \sqsubseteq A$'

DisjointClasses$(C_1, C_2, \dots, C_n)$

$$C_i \sqcap C_j \sqsubseteq \bot, \quad \text{for all } i, j \text{ with } 1 \leq i < j \leq n$$

DisjointUnion$(C, C_1, C_2, \dots, C_n)$,

$$C \equiv C_1 \sqcup \cdots \sqcup C_n$$

SymmetricObjectProperty(P)

$$P^- \sqsubseteq P$$

DL Zoo

$\mathcal{ALCHI}$

$\mathcal{AL}$ – attributive language

$\mathcal{C}$ – complement $\neg C$

$\mathcal{I}$ – role inverses P^-

$\mathcal{H}$ – role inclusions $R_1 \sqsubseteq R_2$

$\mathcal{S}$ – $\mathcal{ALC}$ + transitive roles

$\mathcal{N}$ – unqualified number restrictions $\geq q\, R.\top$

$\mathcal{O}$ – nominals $\{a\}$

$\mathcal{SHOIN} \approx$ OWL 1.0

$\mathcal{Q}$ – qualified number restrictions $\geq q\, R.C$

$\mathcal{F}$ – functionality constraints $\geq 2\, R.\top \sqsubseteq \bot$

$\mathcal{SHIF} \approx$ OWL Lite

$\mathcal{R}$ – role chains and $\exists R.$Self

$\mathcal{SROIQ} \approx$ OWL 2

Complexity of Reasoning

The satisfiability problem is **ExpTime**-complete for $\mathcal{ALCHI}$ KBs
and **N2ExpTime**-complete for $\mathcal{SROIQ}$ KBs

Concept and role subsumption and instance checking are **ExpTime**- and **coN2ExpTime**-complete for, respectively, $\mathcal{ALCHI}$ and $\mathcal{SROIQ}$ KBs

CQ entailment over $\mathcal{ALCHI}$ KBs is **2ExpTime**-complete

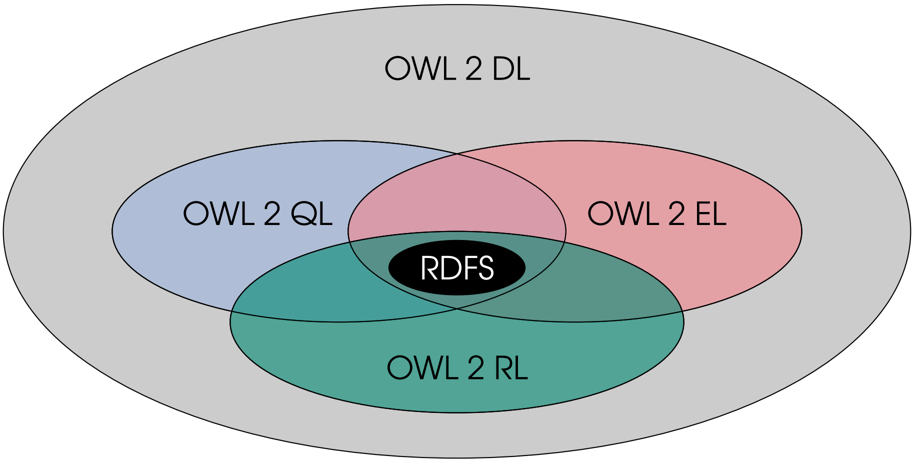
CQ entailment over $\mathcal{SROIQ}$ is not even known to be decidable

DL Complexity Navigator: `www.cs.man.ac.uk/~ezolin/dl`

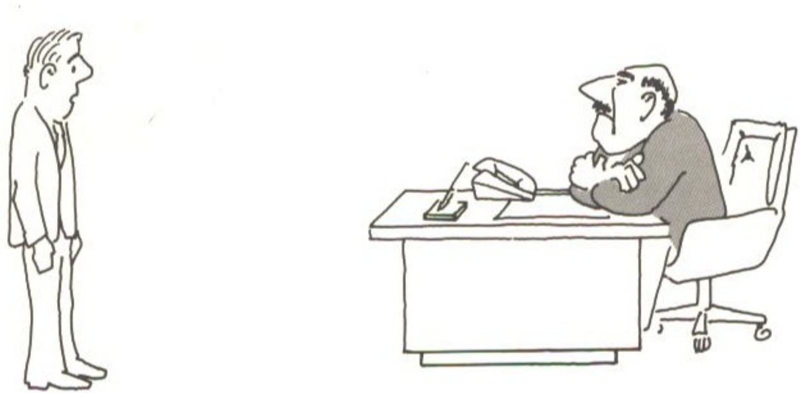
practical reasoners for OWL 2 DL: FaCT++, HermiT, Pellet

Part 2

OWL 2 Profiles



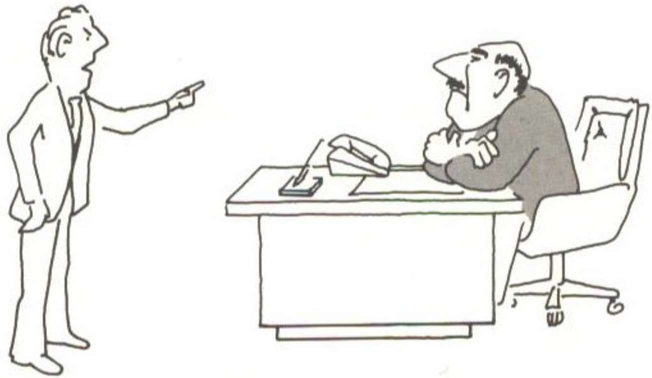
(In)tractable Reasoning



"I can't find an efficient algorithm, I guess I'm just too dumb."

Garey & Johnson, 1979
Computers and Intractability: A Guide to the Theory of NP-Completeness

(In)tractable Reasoning



"I can't find an efficient algorithm, because no such algorithm is possible!"

Garey & Johnson, 1979
Computers and Intractability: A Guide to the Theory of NP-Completeness

(In)tractable Reasoning

"I can't find an efficient algorithm, but neither can all these famous people."

Garey & Johnson, 1979
Computers and Intractability: A Guide to the Theory of NP-Completeness

OWL 2 Profiles: No Disjunction

graph 3-colourability problem is **NP**-complete

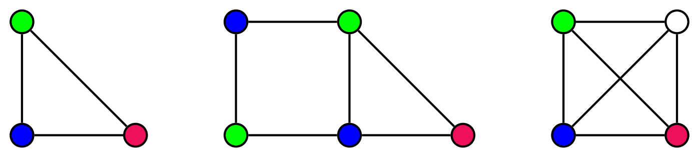
given an (undirected) graph $G = (V, E)$,
decide whether each of its vertices can be painted in one of the three colours
in such a way that no pair of adjacent vertices has the same colour

$\mathcal{A}_G$: edge(v_1, v_2), for each $\{v_1, v_2\} \in E$

$\mathcal{T}$:

$$\boxed{\begin{array}{ll} \top \sqsubseteq C_1 \sqcup C_2 \sqcup C_3 & \\ C_i \sqcap C_j \sqsubseteq \bot, & 1 \leq i < j \leq 3, \\ C_i \sqcap \exists \text{edge}.C_i \sqsubseteq \bot, & 1 \leq i \leq 3 \end{array}}$$

models of $(\mathcal{T}, \mathcal{A}_G)$ $\sim$
3-colouring of G



$(\mathcal{T}, \mathcal{A}_G)$ is satisfiable $\iff$ G is 3-colourable

the satisfiability problem for KBs in any DL able to express this TBox
is **NP**-hard (intractable)

OWL 2 RL

Description Logic Programs (Grosof, Horrocks, Volz, Decker ’03) and pD* (ter Horst ’05)

supported by Oracle Database 11g, OWLIM, BaseVISor, ELLY, Jena and RDFox

RL TBox:

$$\boldsymbol{B \sqsubseteq A}, \qquad \boldsymbol{R_1 \sqsubseteq R_2} \quad \text{and} \quad \boldsymbol{B \sqsubseteq \bot}$$

only concept names on the right

$$\boldsymbol{B} \quad ::= \quad \boldsymbol{A} \quad | \quad \exists \boldsymbol{R}.\top \quad | \quad \exists \boldsymbol{R}.\boldsymbol{B} \quad | \quad \boldsymbol{B_1} \sqcap \boldsymbol{B_2}$$

simple ABox:

$\boldsymbol{A(a)}$ and $\boldsymbol{P(a, b)}$

NB: this is a simplified version — see the equivalences

RL and Datalog

UndergraduateStudent $\sqsubseteq$ Student

$$\forall x\,(\mathsf{UndergraduateStudent}(x) \to \mathsf{Student}(x))$$

$\exists$takesCourse.UndergraduateCourse $\sqsubseteq$ UndergraduateStudent

$$\forall x \forall y\,(\mathsf{takesCourse}(x, y) \land \mathsf{UndergraduateCourse}(y) \\ \to \mathsf{UndergraduateStudent}(x))$$

standard translation $\quad ST_x(A) = A(x)$ and $ST_{x,y}(P) = P(x, y)$

$$ST_{x,y}(P^-) = P(y, x) \qquad ST_x(B_1 \sqcap B_2) = ST_x(B_1) \land ST_x(B_2)$$
$$ST_x(\exists R.\top) = \exists y\, ST_{x,y}(R) \qquad ST_x(\exists R.B) = \exists y\,(ST_{x,y}(R) \land ST_y(B))$$

$$(B \sqsubseteq A)^* = \forall x\,(ST_x(B) \to ST_x(A))$$
$$(R_1 \sqsubseteq R_2)^* = \forall x \forall y\,(ST_{x,y}(R_1) \to ST_{x,y}(R_2))$$
$$(B \sqsubseteq \bot)^* = \forall x\,(ST_x(B) \to \bot)$$

$$\forall \vec{y}\,(\gamma_1(\vec{y}) \land \cdots \land \gamma_k(\vec{y}) \to \gamma_0(\vec{y}))$$
$$\forall \vec{y}\,(\gamma_1(\vec{y}) \land \cdots \land \gamma_k(\vec{y}) \to \bot)$$

Horn clauses (datalog programs)

Chase: Example

$\mathcal{T}$: $\exists$takesCourse.UndergraduateCourse $\sqsubseteq$ UndergraduateStudent
UndergraduateStudent $\sqsubseteq$ Student

$\mathcal{A}$: takesCourse(john, sp1), UndergraduateCourse(sp1)

step 0: $\mathcal{I}_0 \approx \mathcal{A}$

$$\begin{aligned}
&\Delta^{\mathcal{I}_0} = \{\text{john}, \text{sp1}\}\\
&\text{takesCourse}^{\mathcal{I}_0} = \{(\text{john}, \text{sp1})\}\\
&\text{UndergraduateCourse}^{\mathcal{I}_0} = \{\text{sp1}\}\\
&\text{UndergraduateStudent}^{\mathcal{I}_0} = \emptyset\\
&\text{Student}^{\mathcal{I}_0} = \emptyset
\end{aligned}$$

$\mathcal{I}_0 \not\models \mathcal{T}$

step 1: $\Delta^{\mathcal{I}_1} = \Delta^{\mathcal{I}_0}$ and $\cdot^{\mathcal{I}_1}$ expands $\cdot^{\mathcal{I}_0}$ with

$$\text{UndergraduateStudent}^{\mathcal{I}_1} = \{\text{john}\}$$

$\mathcal{I}_1 \not\models \mathcal{T}$

step 2: $\Delta^{\mathcal{I}_2} = \Delta^{\mathcal{I}_1}$ and $\cdot^{\mathcal{I}_1}$ expands $\cdot^{\mathcal{I}_1}$ with

$$\text{Student}^{\mathcal{I}_2} = \{\text{john}\}$$

$\mathcal{I}_2 \models \mathcal{T}$

stop: $\mathcal{I}_2$ is a model of $(\mathcal{T}, \mathcal{A})$

Chase for RL

the **standard model** $\mathcal{I}_\mathcal{A}$ of a simple ABox $\mathcal{A}$ is

$$
\begin{aligned}
\Delta^{\mathcal{I}_\mathcal{A}} &= \text{ind}(\mathcal{A})\\
a^{\mathcal{I}_\mathcal{A}} &= a, \ \text{ for } a \in \text{ind}(\mathcal{A})\\
A^{\mathcal{I}_\mathcal{A}} &= \{a \mid A(a) \in \mathcal{A}\}, \ \text{ for concept name } A\\
P^{\mathcal{I}_\mathcal{A}} &= \{(a, b) \mid P(a, b) \in \mathcal{A}\}, \ \text{ for role name } P
\end{aligned}
$$

chase rules: (**forward chaining**)

- **(c)** if $a \in B^{\mathcal{I}_k}$, $B \sqsubseteq A \in \mathcal{T}$ but $a \notin A^{\mathcal{I}_k}$, then add a to $A^{\mathcal{I}_{k+1}}$
- **(r)** if $(a, b) \in R_1^{\mathcal{I}_k}$, $R_1 \sqsubseteq R_2 \in \mathcal{T}$ but $(a, b) \notin R_2^{\mathcal{I}_k}$, then add (a, b) to $R_2^{\mathcal{I}_{k+1}}$
- **(b)** if $a \in B^{\mathcal{I}_k}$ and $B \sqsubseteq \bot \in \mathcal{T}$, then the process terminates

the domains of the $\mathcal{I}_k$ are finite and all coincide with ind$(\mathcal{A})$,
$\Rightarrow$ the process **terminates** after a finite number of steps

– if **(b)** is not applicable then the result is a model of $(\mathcal{T}, \mathcal{A})$,
the **canonical model** $\mathcal{C}_{\mathcal{T},\mathcal{A}}$ of $(\mathcal{T}, A)$

– otherwise, $(\mathcal{T}, \mathcal{A})$ is inconsistent

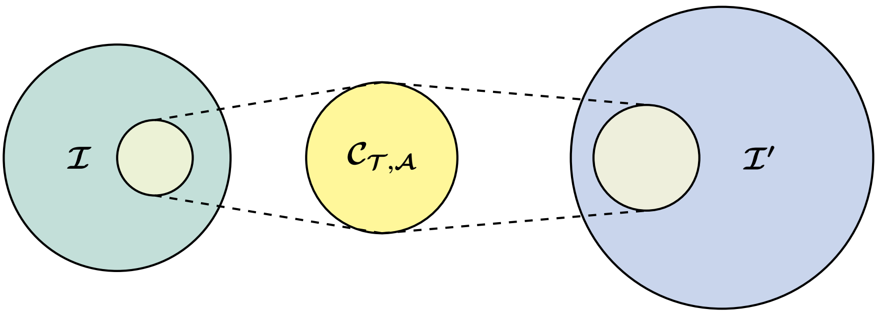
Canonical Model is Universal

a **homomorphism** h from an interpretation $\mathcal{I}_1$ to an interpretation $\mathcal{I}_2$ is a map from $\Delta^{\mathcal{I}_1}$ to $\Delta^{\mathcal{I}_2}$ such that

- $h(a^{\mathcal{I}_1}) = a^{\mathcal{I}_2}$, for each individual name a
- $h(u) \in A^{\mathcal{I}_2}$, for any $u \in A^{\mathcal{I}_1}$ and any concept name A
- $(h(u), h(v)) \in P^{\mathcal{I}_2}$, for any $(u, v) \in P^{\mathcal{I}_1}$ and any role name P

Lemma $\mathcal{C}_{\mathcal{T},\mathcal{A}}$ is **universal**

$\mathcal{C}_{\mathcal{T},\mathcal{A}}$ can be homomorphically mapped into any other model of $(\mathcal{T}, \mathcal{A})$



Complexity of Reasoning in RL

Theorem Let $(\mathcal{T}, \mathcal{A})$ be a consistent RL KB. Then $\mathcal{C}_{\mathcal{T},\mathcal{A}} \models (\mathcal{T}, \mathcal{A})$. In addition,

$$(\mathcal{T}, \mathcal{A}) \models B \sqsubseteq A \iff \mathcal{C}_{\mathcal{T},\mathcal{A}} \models B \sqsubseteq A$$

$$(\mathcal{T}, \mathcal{A}) \models R_1 \sqsubseteq R_2 \iff \mathcal{C}_{\mathcal{T},\mathcal{A}} \models R_1 \sqsubseteq R_2$$

$$(\mathcal{T}, \mathcal{A}) \models q(\vec{a}) \iff \mathcal{C}_{\mathcal{T},\mathcal{A}} \models q(\vec{a})$$

Theorem The problems of KB consistency, concept and role subsumption and instance checking are **P**-complete in RL

The problem of CQ entailment in RL is **NP**-complete

Proof Construct the canonical model (in a polynomial number of steps).

Check the condition in the canonical model.

OWL 2 EL

SNOMED CT (Spackman '00, Baader, Brandt & Lutz '05)

Pericardium $\sqsubseteq$ Tissue $\sqcap$ containedIn.Heart
Pericarditis $\sqsubseteq$ Inflammation $\sqcap$ hasLocation.Pericardium
Inflammation $\sqsubseteq$ Disease $\sqcap$ actsOn.Tissue
Disease $\sqcap$ hasLocation.containedIn.Heart $\sqsubseteq$ HeartDisease $\sqcap$ NeedsTreatment

EL TBox:

$C_1 \sqsubseteq C_2$ and $P_1 \sqsubseteq P_2$

$$C \quad ::= \quad A \quad | \quad \exists P.\top \quad | \quad \exists P.C \quad | \quad C_1 \sqcap C_2$$

no inverses

simple ABox:

$A(a)$ and $P(a, b)$

EL and Existential Rules

Person $\sqsubseteq$ $\exists$parent.Person

$$\forall x\,(\text{Person}(x) \to \exists y\,(\text{parent}(x, y) \land \text{Person}(y)))$$

the standard translation can be extended to EL

$$\forall \vec{y}\,(\gamma_1(\vec{y}) \land \dots \land \gamma_k(\vec{y}) \to \exists \vec{x}\,\gamma_0(\vec{x}, \vec{y}))$$

existential rules

chase rules: (Johnson & Klug '84)

(c′) if $d \in C^{\mathcal{I}_k}$ and $C \sqsubseteq A \in \mathcal{T}$, then add d to $A^{\mathcal{I}_{k+1}}$

(r′) if $(d, d') \in P_1^{\mathcal{I}_k}$ and $P_1 \sqsubseteq P_2 \in \mathcal{T}$, then we add (d, d') to $P_2^{\mathcal{I}_{k+1}}$

(e) if $d \in C^{\mathcal{I}_k}$ and $C \sqsubseteq \exists P.D \in \mathcal{T}$, where D is a concept name or $\top$,
then take a **fresh labelled null**, d', and add d' to $D^{\mathcal{I}_{k+1}}$ and (d, d') to $P^{\mathcal{I}_{k+1}}$

NB: oblivious chase — no check whether there is such an element

EL Chase: Example

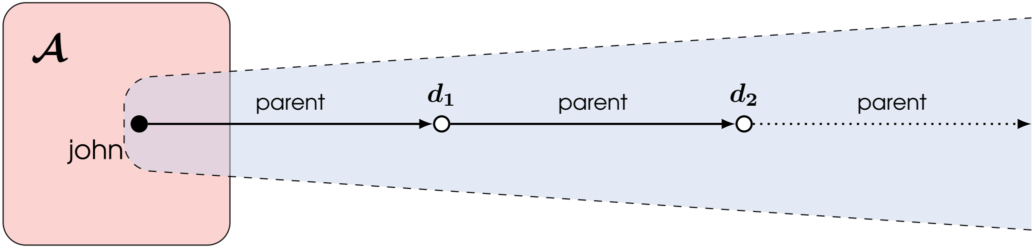
$\mathcal{T}$: Person $\sqsubseteq$ $\exists$parent.Person

$\mathcal{A}$: Person(john)

step 0: $\Delta^{\mathcal{I}_0} = \{\text{john}\}$, $\text{Person}^{\mathcal{I}_0} = \{\text{john}\}$, $\text{parent}^{\mathcal{I}_0} = \emptyset$ $\quad \mathcal{I}_0 = \mathcal{I}_\mathcal{A}$

step 1: $\Delta^{\mathcal{I}_1} = \{\text{john}, d_1\}$, $\text{Person}^{\mathcal{I}_1} = \{\text{john}, d_1\}$, $\text{parent}^{\mathcal{I}_1} = \{(\text{john}, d_1)\}$

step 2: $\Delta^{\mathcal{I}_2} = \{\text{john}, d_1, d_2\}$, $\text{Person}^{\mathcal{I}_2} = \{\text{john}, d_1, d_2\}$,
$\text{parent}^{\mathcal{I}_2} = \{(\text{john}, d_1), (d_1, d_2)\}$



1. this canonical model is infinite
2. there is no finite universal model
3. there are many universal models (each is infinite)

Normal Form in EL

normal form of concept inclusions

$$\boxed{A_1 \sqcap A_2 \sqsubseteq A, \qquad \exists P.D \sqsubseteq A \quad \text{or} \quad A \sqsubseteq \exists P.D}$$

P is a role name, A, A_1 and A_2 are concept names and D is either a concept name or $\top$

Example: $\exists P.A \sqcap B \sqsubseteq \exists R.\exists P.A$

introduce abbreviations: C is $\exists P.A$ and D is $\exists R.C$

result:

$$C \sqcap B \sqsubseteq D, \qquad \exists P.A \sqsubseteq C, \qquad C \sqsubseteq \exists P.A, \qquad D \sqsubseteq \exists R.C, \qquad \exists R.C \sqsubseteq D$$

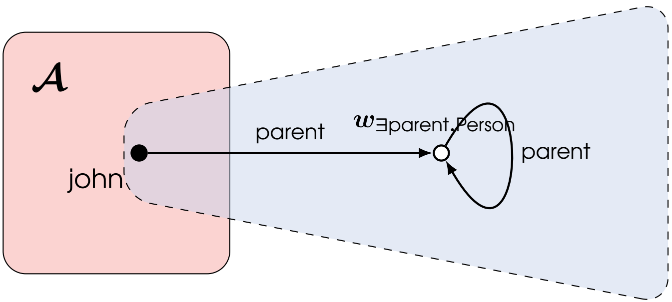
Reasoning in EL

do not construct the infinite chase, **re-use** the labelled nulls

for each positive $\exists S.D$, take a fixed **witness** $w_{\exists S.D}$

(e-c) if $d \in C^{\mathcal{I}_k}$ and $C \sqsubseteq \exists P.D \in \mathcal{T}$, where D is a concept name or $\top$,

then add $w_{\exists S.D}$ to $D^{\mathcal{I}_{k+1}}$ and (d, d') to $P^{\mathcal{I}_{k+1}}$



the **generating model** $\mathcal{G}_{\mathcal{T},\mathcal{A}}$ of $(\mathcal{T}, \mathcal{A})$

NB: alternatively,

take the canonical model and identify all labelled nulls introduced for the same $\exists S.D$

Reasoning in EL (2)

Theorem Let $(\mathcal{T},\mathcal{A})$ be an EL KB. Then $\mathcal{G}_{\mathcal{T},\mathcal{A}} \models (\mathcal{T},\mathcal{A})$. In addition,

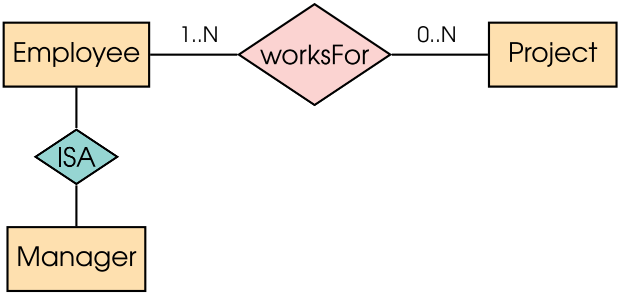
$$(\mathcal{T},\mathcal{A}) \models C_1 \sqsubseteq C_2 \iff \mathcal{G}_{\mathcal{T},\mathcal{A}} \models C_1 \sqsubseteq C_2$$
$$(\mathcal{T},\mathcal{A}) \models P_1 \sqsubseteq P_2 \iff \mathcal{G}_{\mathcal{T},\mathcal{A}} \models P_1 \sqsubseteq P_2$$
$$(\mathcal{T},\mathcal{A}) \models C(a) \iff \mathcal{G}_{\mathcal{T},\mathcal{A}} \models C(a)$$
$$(\mathcal{T},\mathcal{A}) \models P(a,b) \iff \mathcal{G}_{\mathcal{T},\mathcal{A}} \models P(a,b)$$

Theorem The problems of concept and role subsumption and instance checking are **P**-complete in EL

NB: $\mathcal{G}_{\mathcal{T},\mathcal{A}}$ does **not** give correct answers to queries

$$q = \exists x\, \text{parent}(x,x)$$

OWL 2 QL



(Calvanese et al. '05)

$\exists \text{worksFor}.\top \sqsubseteq \text{Employee}$
$\exists \text{worksFor}^-.\top \sqsubseteq \text{Project}$
$\text{Manager} \sqsubseteq \text{Employee}$
$\text{Project} \sqsubseteq \exists \text{worksFor}^-.\top$

QL TBox:

$\boldsymbol{B} \sqsubseteq \boldsymbol{C}$ and $\boldsymbol{R_1} \sqsubseteq \boldsymbol{R_2}$

$$\begin{array}{lllllll} \boldsymbol{B} & ::= & \boldsymbol{A} & \mid & \exists \boldsymbol{R}.\top & & \\ \boldsymbol{C} & ::= & \boldsymbol{A} & \mid & \exists \boldsymbol{R}.\top & \mid & \exists \boldsymbol{R}.\boldsymbol{C} \end{array}$$

no $\exists R.C$ on the left

simple ABox:

$\boldsymbol{A}(\boldsymbol{a})$ and $\boldsymbol{P}(\boldsymbol{a}, \boldsymbol{b})$

Chase for QL

normal form of concept inclusions

$$\boxed{A' \sqsubseteq A, \qquad \exists R \sqsubseteq A \quad \text{or} \quad A \sqsubseteq \exists R.D}$$

R is a role, A and A' are concept names and D is either a concept name or $\top$

chase rules:

(c′) if $d \in B^{\mathcal{I}_k}$ and $B \sqsubseteq A \in \mathcal{T}$, then add d to $A^{\mathcal{I}_{k+1}}$

(r′) if $(d, d') \in R_1^{\mathcal{I}_k}$ and $R_1 \sqsubseteq R_2 \in \mathcal{T}$, then add (d, d') to $R_2^{\mathcal{I}_{k+1}}$

(e) if $d \in B^{\mathcal{I}_k}$ and $B \sqsubseteq \exists R.D \in \mathcal{T}$, where D is a concept name or $\top$,
then take a **fresh labelled null**, d', and add d' to $D^{\mathcal{I}_{k+1}}$ and (d, d') to $R^{\mathcal{I}_{k+1}}$

Unravelling of the Generating Structure

for each $\exists R.D$ that occurs positively in $\mathcal{T}$, take a **witness** $w_{\exists R.D}$

generating relation $\leadsto_{\mathcal{T},\mathcal{A}}$:

$a \leadsto_{\mathcal{T},\mathcal{A}} w_{\exists R.D}$	if	$a \in \mathsf{ind}(\mathcal{A})$, $\mathcal{I}_{\mathcal{A}} \models B(a)$ and $\mathcal{T} \models B \sqsubseteq \exists R.D$
$w_{\exists S.B} \leadsto_{\mathcal{T},\mathcal{A}} w_{\exists R.D}$	if	$\mathcal{T} \models \exists S^{-} \sqsubseteq \exists R.D$ or $\mathcal{T} \models B \sqsubseteq \exists R.D$

$\leadsto_{\mathcal{T},\mathcal{A}}$**-path** σ is any $a w_{\exists R_1.D_1} \cdots w_{\exists R_n.D_n}$

– $a \in \mathsf{ind}(\mathcal{A})$ and,

– if $n > 0$ then $a \leadsto_{\mathcal{T},\mathcal{A}} w_{\exists R_1.D_1}$ and $w_{\exists R_i.D_i} \leadsto_{\mathcal{T},\mathcal{A}} w_{\exists R_{i+1}.D_{i+1}}$, for $i < n$

Canonical Model: Example

$\mathcal{T}$: RA $\sqsubseteq \exists$worksOn.Project   worksOn$^- \sqsubseteq$ involves

Project $\sqsubseteq \exists$isManagedBy.Prof   isManagedBy $\sqsubseteq$ involves

$\mathcal{A}$: RA(chris), worksOn(chris, dyn), Project(dyn), Lecturer(dave), worksOn(dave, dyn)

$w_1 = w_{\exists \text{worksOn.Project}}$ and $w_2 = w_{\exists \text{isManagedBy.Prof}}$

generating relation

chris $\leadsto_{\mathcal{T},\mathcal{A}} w_1$,   dyn $\leadsto_{\mathcal{T},\mathcal{A}} w_2$, $w_1 \leadsto_{\mathcal{T},\mathcal{A}} w_2$

$\leadsto_{\mathcal{T},\mathcal{A}}$-paths

chris,   chris w_1,   chris w_1 w_2,   dyn,   dyn w_2 and dave

Canonical Model: Example (2)

$\mathcal{T}$:

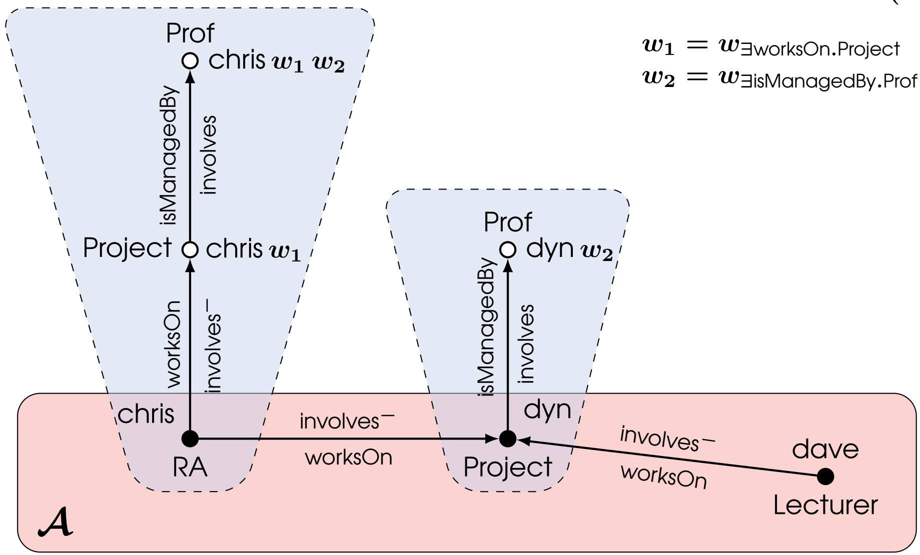
$\mathsf{RA} \sqsubseteq \exists \mathsf{worksOn}.\mathsf{Project}$ $\qquad$ $\mathsf{worksOn}^- \sqsubseteq \mathsf{involves}$

$\mathsf{Project} \sqsubseteq \exists \mathsf{isManagedBy}.\mathsf{Prof}$ $\qquad$ $\mathsf{isManagedBy} \sqsubseteq \mathsf{involves}$

$\mathcal{A}$: RA(chris), worksOn(chris, dyn), Project(dyn), Lecturer(dave), worksOn(dave, dyn)

$w_1 = w_{\exists \mathsf{worksOn}.\mathsf{Project}}$

$w_2 = w_{\exists \mathsf{isManagedBy}.\mathsf{Prof}}$



Comparing Profiles

	RL	EL	QL
inverse roles P^-	+	–	+
$\exists R.C$ on the right	–	+	+
$\exists R.C$ on the left	+	+	–

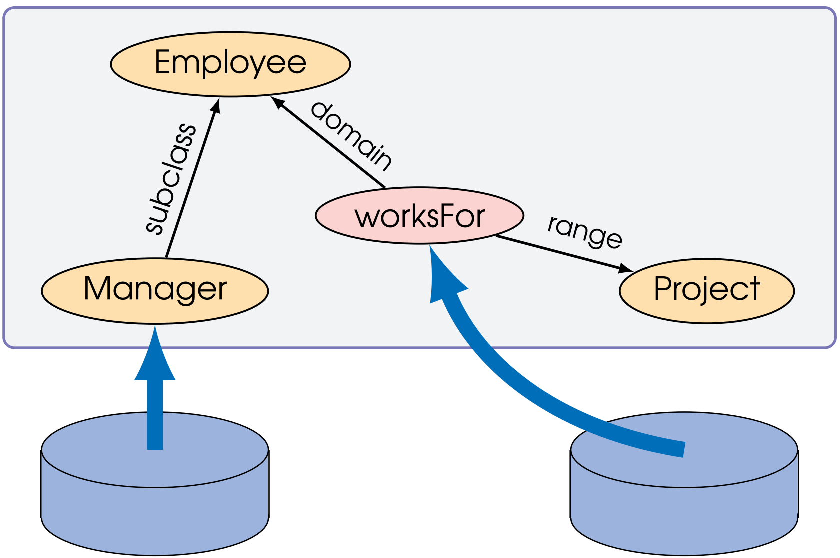
Part 3

OBDA and Query Rewriting

Ontology-Based Data Access

Aim: to achieve **logical transparency** in accessing data

- hide from the user where and how data is stored
- present only a **conceptual view** of the data
- **query** the data sources through the **conceptual model** using **RDBMSs**



ontology
intensional level,
conceptual structure of
the domain,
TBox

(GAV) mappings

data sources
extensional level,
instances of conceptual
elements,
ABox

Databases: Data and the Closed World Assumption

data is **completely** specified (**closed world assumption**) and is typically **large**

what is specified is true, everything else is false

data: $\mathsf{Employee} = \{\mathsf{john}, \mathsf{mary}, \mathsf{nick}\}$
$\mathsf{Manager} = \{\mathsf{john}, \mathsf{nick}\}$
$\mathsf{Project} = \{\mathsf{A}, \mathsf{B}\}$
$\mathsf{worksFor} = \{(\mathsf{john}, \mathsf{A}), (\mathsf{mary}, \mathsf{B})\}$

query: $q(x) \leftarrow \mathsf{Employee}(x)$

answer: $\{\mathsf{john}, \mathsf{mary}, \mathsf{nick}\}$

NB: not having nick in Employee would violate the integrity constraint

$$\forall x\ (\mathsf{Manager}(x) \rightarrow \mathsf{Employee}(x))$$

Why do Databases Work?

query answering problem (as a recognition problem):

given a finite data $\mathcal{D}$, a query $q(\vec{x})$ and a tuple $\vec{a}$,
decide whether $\mathcal{I}_{\mathcal{D}} \models q(\vec{a})$

$\mathcal{I}_{\mathcal{D}}$ makes the facts in $\mathcal{D}$ true (and only them)

what is the complexity of CQ answering?

naive algorithm:

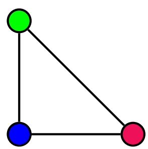
guess values for all existential variables and then
evaluate the query in polynomial time — in **NP**

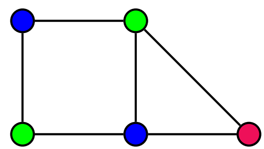
can it be done better?

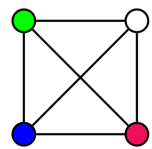
Why Do Databases Work? (2)

no, by reduction of the graph 3-colourability problem, which is **NP-complete**:

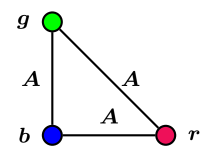
'given an undirected graph $G = (V, E)$,
decide whether it possible to colour it (using r, g, b)
so that no edge has the same colour at both ends?'







$$
\begin{aligned}
\mathcal{D} &= \{A(r,g), A(g,b), A(b,r), A(g,r), A(r,b), A(b,g)\}\\
q_G &= \exists v_1, \dots, v_n \bigwedge_{(v_i,v_j)\in E} A(v_i, v_j)
\end{aligned}
$$



$$\boxed{\mathcal{D} \models q_G \quad \text{iff} \quad G \text{ is 3-colourable}}$$

in fact, the query answering algorithm runs in $O(|\mathcal{D}|^{|q|})$ **data is large, query is short**

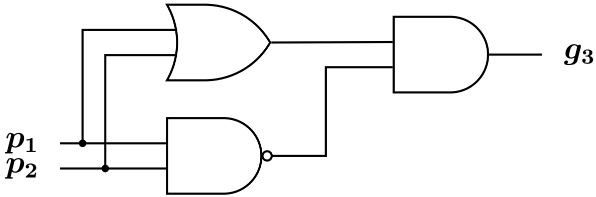
data complexity: only data $\mathcal{D}$ are counted as **input** (q is constant)

(Vardi, 1982): query answering is in AC^0 for data complexity

Circuits and AC^0

a **circuit** is an acyclic graph of AND-, OR- and NOT-gates

(with n inputs and a single output, **sink**)



database instances $\mathcal{D}$ can be encoded on inputs

(one input for each possible ground atom)

FO-query is a circuit: $\land$, $\lor$ and $\neg$ are AND-, OR- and NOT-gates, respectively

$\forall$ and $\exists$ are AND- and OR-gates with unbounded fan-in

AC^0 = circuits of constant depth with AND- and OR-nodes of unbounded fan-in

constant time by a polynomial number of processors (high degree of parallelism)

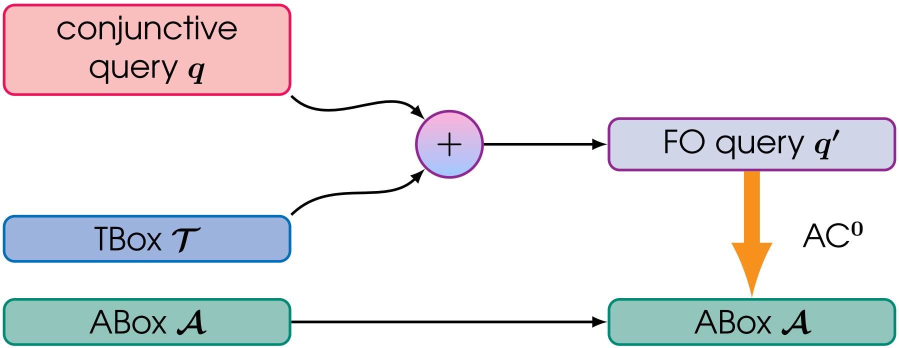
the depth of this circuit does not depend on $\mathcal{D}$ $\longrightarrow$ Vardi's theorem

NB: AC^0 is a **proper** subclass of LogSpace $\subseteq$ P (Parity does not belong to AC^0)

given a word w, decide whether its length is even

Query Rewriting Approach

use off-the-shelf RDBMS (Calvanese et al. ’05)



given a CQ $q(\vec{x})$ over $\mathcal{T}$, rewrite $q(\vec{x})$ into an FO query $q'(\vec{x})$ such that

for all $\mathcal{A}$ and $\vec{a}$, $\mathcal{T}, \mathcal{A} \models q(\vec{a})$ iff $\mathcal{A} \models q'(\vec{a})$

FO-rewritability: only possible in DL with query answering in **FO** (=AC^0) for data complexity:

OWL 2 QL

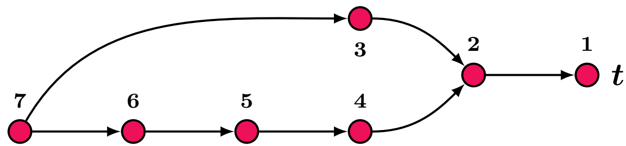
Can We Use $\exists R.A \sqsubseteq B$ in QL?

reachability problem for directed graphs is **NLogSpace-complete**:

'given a directed graph $G = (V, E)$ and $s, t \in V$, decide whether there is a directed path from s to t in G'

ABox: $\mathcal{A}_{G,t} = \{\, \mathsf{edge}(v_1, v_2) \mid (v_1, v_2) \in E \,\} \cup \{\mathsf{ReachableFromTarget}(t)\}$

TBox: $\mathcal{T} = \{\, \exists \mathsf{edge}.\mathsf{ReachableFromTarget} \sqsubseteq \mathsf{ReachableFromTarget} \,\}$



CQ: $q \leftarrow \mathsf{ReachableFromTarget}(s)$

$(\mathcal{T}, \mathcal{A}_{G,t}) \models q$ iff there is a path from s to t in G

$\mathcal{T}$ and q do not depend on G, s, t

→ '$(\mathcal{T}, \mathcal{A}_{G,t}) \models q$?' is **NLogSpace**-hard for data complexity

→ q and $\mathcal{T}$ are not FO-rewritable

Practical Query Answering in OWL 2 QL

systems

- QuOnto (Rome, 2005)
- REQUIEM (Oxford, 2009)
- Presto (Rome, 2010)
- IQAROS (Athens, 2011)
- Rapid (Athens-Oxford, 2011)
- Nyaya (Milan-Oxford, 2010) for TGDs
- Clipper (Vienna, 2012) for Horn-SHIQ
- (Montpellier, 2013) for TGDs
- Quest/ontop (Bolzano, 2011)

not so smoothly: the size of implemented rewritings q' is $O\big((|q| \cdot |\mathcal{T}|)^{|q|}\big)$

(can't say 'query is small or fixed' any longer)

Does a Rewriting Have to be Exponential?

TBox $\boxed{\text{mother} \sqsubseteq \text{parent} \quad \text{and} \quad \text{father} \sqsubseteq \text{parent}}$

query $\boxed{\text{grandparent}(x, z) \leftarrow \text{parent}(x, y) \wedge \text{parent}(y, z)}$

UCQ-rewritings (unions of CQs) are **exponential** in the worst case

$\text{grandparent}(x, z) \leftarrow \text{parent}(x, y) \wedge \text{parent}(y, z)$
$\text{grandparent}(x, z) \leftarrow \text{father}(x, y) \wedge \text{father}(y, z)$
$\text{grandparent}(x, z) \leftarrow \text{mother}(x, y) \wedge \text{father}(y, z)$
$\dots$

PE-rewritings (positive existential queries ≈ select-project-join-union) $\boxed{\exists \ \vee \ \wedge}$

$$\text{grandparent}(x, z) \leftarrow (\text{parent}(x, y) \vee \text{father}(x, y) \vee \text{mother}(x, y)) \wedge \\ (\text{parent}(y, z) \vee \text{father}(y, z) \vee \text{mother}(y, z))$$

NDL-rewriting (non-recursive Datalog ≈ SQL with views) $\boxed{\exists \ \vee \ \wedge + \text{structure sharing}}$

$\text{grandparent}(x, z) \leftarrow \text{ext-parent}(x, y) \wedge \text{ext-parent}(y, z)$
$\text{ext-parent}(x, y) \leftarrow \text{parent}(x, y)$
$\text{ext-parent}(x, y) \leftarrow \text{father}(x, y)$
$\text{ext-parent}(x, y) \leftarrow \text{mother}(x, y)$

FO-rewriting (first-order queries ≈ SQL) $\boxed{\exists \forall \ \vee \ \wedge \neg}$

Case 1: Flat QL TBoxes

a TBox $\mathcal{T}$ is **flat** if it does not contain **generating axioms** $B' \sqsubseteq \exists R.B$

$\approx$ RDF Schema

$q(\vec{x})$ and a flat $\mathcal{T}$ $\longrightarrow$ $q_{\text{ext}}(\vec{x})$ by replacing

$$A(u) \longrightarrow \bigvee_{\mathcal{T} \models A' \sqsubseteq A} A'(u) \lor \bigvee_{\mathcal{T} \models \exists R \sqsubseteq A} \exists v\, R(u,v)$$

$$P(u,v) \longrightarrow \bigvee_{\mathcal{T} \models R \sqsubseteq P} R(u,v)$$

for any CQ $q(\vec{x})$ and any flat OWL 2 QL TBox $\mathcal{T}$,

$q_{\text{ext}}(\vec{x})$ is a PE-rewriting of q and $\mathcal{T}$ of size $O(|q| \cdot |\mathcal{T}|)$

easy in theory, not so in practice

Who Works with Professors?

TBox:

$$\mathsf{worksOn}^- \sqsubseteq \mathsf{involves}$$
$$\mathsf{isManagedBy} \sqsubseteq \mathsf{involves}$$

in English: find those who work with professors

query: $q(x) \leftarrow \mathsf{worksOn}(x, y) \wedge \mathsf{involves}(y, z) \wedge \mathsf{Professor}(z)$

$\mathsf{worksOn}(z, y) \vee \mathsf{isManagedBy}(y, z) \vee \mathsf{involves}(y, z)$

Rewriting over H-complete ABoxes

an ABox $\mathcal{A}$ is **H-complete with respect to $\mathcal{T}$** if

- $A(a) \in \mathcal{A}$ whenever $A'(a) \in \mathcal{A}$ and $\mathcal{T} \models A' \sqsubseteq A$
- $A(a) \in \mathcal{A}$ whenever $R(a, b) \in \mathcal{A}$ and $\mathcal{T} \models \exists R \sqsubseteq A$
- $P(a, b) \in \mathcal{A}$ whenever $R(a, b) \in \mathcal{A}$ and $\mathcal{T} \models R \sqsubseteq P$

an FO-query $q'(\vec{x})$ is an **FO-rewriting of $q(\vec{x})$ and $\mathcal{T}$ over H-complete ABoxes** if,

for any H-complete (w.r.t. $\mathcal{T}$) ABox $\mathcal{A}$ and any $\vec{a}$,

$(\mathcal{T}, \mathcal{A}) \models q(\vec{a})$ iff $\mathcal{A} \models q'(\vec{a})$

(thus we ignore the axioms considered in the flat rewriting)

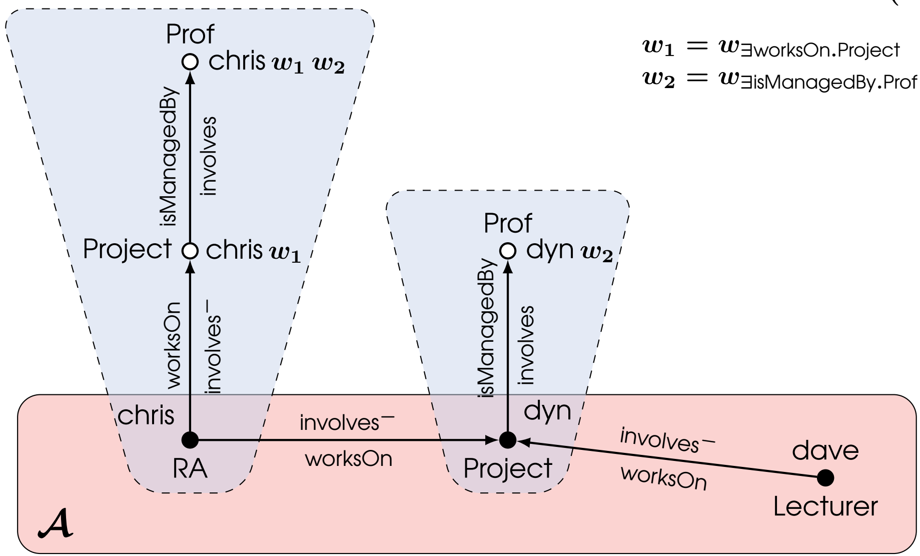
Case 2: Who Works with Professors (2)?

$\mathcal{T}$:

$\mathsf{RA} \sqsubseteq \exists \mathsf{worksOn}.\mathsf{Project}$ $\qquad$ $\mathsf{worksOn}^- \sqsubseteq \mathsf{involves}$

$\mathsf{Project} \sqsubseteq \exists \mathsf{isManagedBy}.\mathsf{Prof}$ $\qquad$ $\mathsf{isManagedBy} \sqsubseteq \mathsf{involves}$

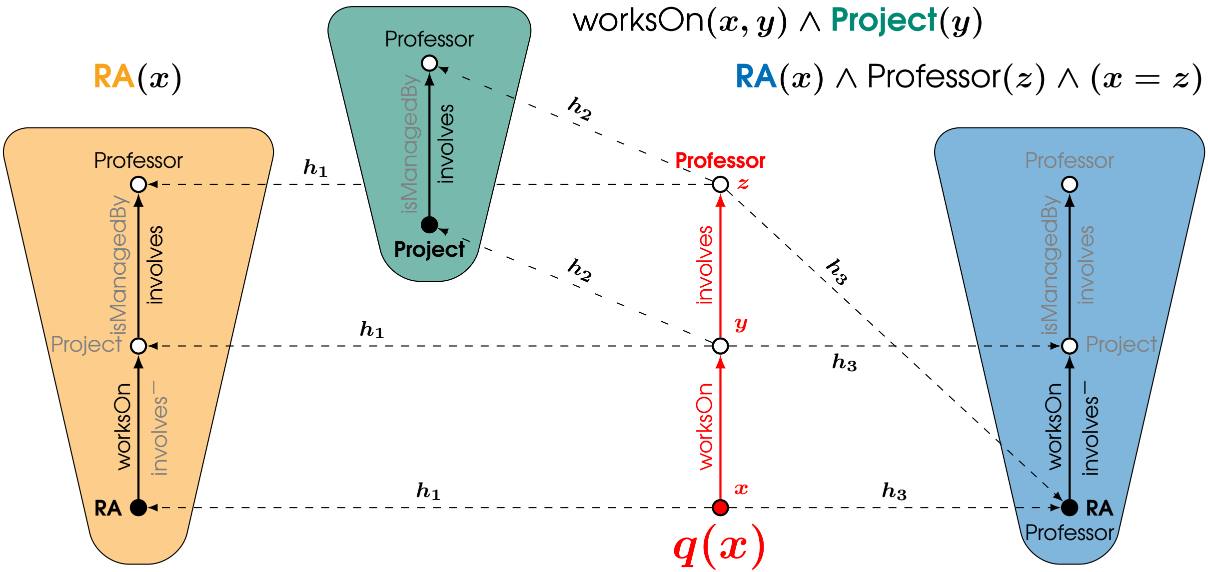
$\mathcal{A}$: RA(chris), worksOn(chris, dyn), Project(dyn), Lecturer(dave), worksOn(dave, dyn)



$w_1 = w_{\exists \mathsf{worksOn}.\mathsf{Project}}$

$w_2 = w_{\exists \mathsf{isManagedBy}.\mathsf{Prof}}$

Case 2: Rewriting the Labelled Nulls



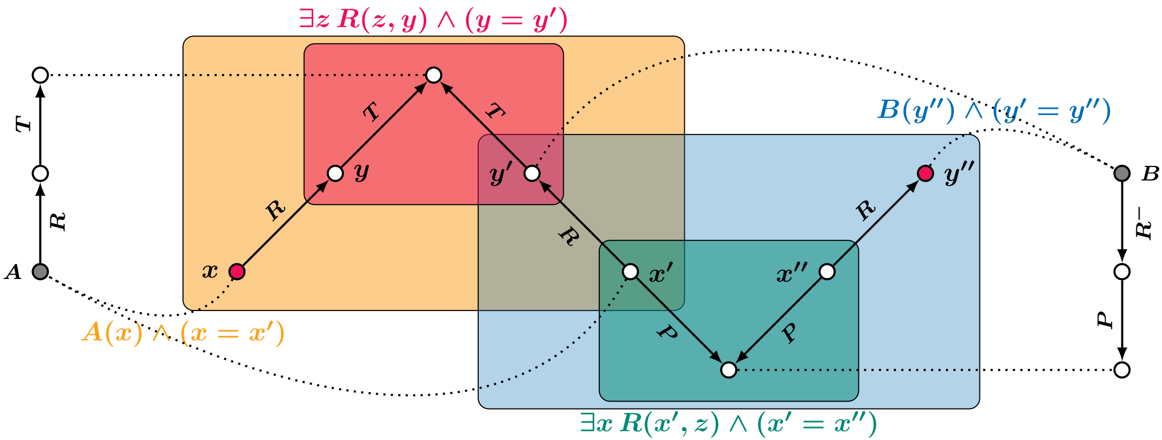
(x and z are the **roots** of the tree witness)

PE-rewriting (over H-complete ABoxes):

$$q'(x) \;\leftarrow\; \mathsf{RA}(x) \;\lor\; (\mathsf{worksOn}(x,y) \land \mathsf{Project}(y)) \;\lor$$
$$(\mathsf{RA}(x) \land \mathsf{Professor}(z) \land (x = z)) \lor$$
$$(\mathsf{worksOn}(x,y) \land \mathsf{involves}(y,z) \land \mathsf{Professor}(z))$$

Tree-Witness Rewriting

TBox $\mathcal{T}$: $\quad A \sqsubseteq \exists R, \quad \exists R^- \sqsubseteq \exists T, \quad B \sqsubseteq \exists R^-, \quad \exists R \sqsubseteq \exists S$



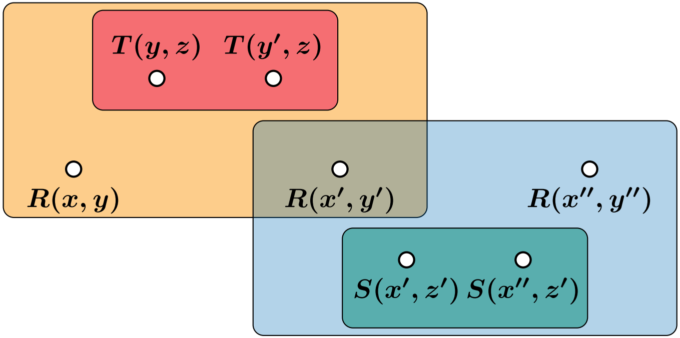
$$q_{\mathsf{tw}}(\vec{x}) \;=\; \bigvee_{\substack{\Theta \text{ independent set}\\ \text{of tree witnesses}}} \exists \vec{y}\Big(\bigwedge_{S(\vec{z}) \in q \setminus q_\Theta} S(\vec{z}) \;\wedge\; \bigwedge_{\mathfrak{t} \in \Theta} \mathsf{tw}_{\mathfrak{t}} \Big)$$

Rewritings as Boolean functions

hypergraph H:

vertices = atoms of the query

hyperedges = tree witnesses



hypergraph function of $H = (V, E)$:

$$f_H = \bigvee_{\substack{X \subseteq E \\ X \text{ independent}}} \Big(\bigwedge_{v \in V \setminus V_X} p_v \wedge \bigwedge_{e \in X} p_e \Big)$$

lower bounds from circuit complexity:

exponential non-recursive datalog (and positive existential) rewritings

superpolynomial first-order rewritings (unless NP $\subseteq$ P/poly)

Short Rewritings in Theory

if $\boldsymbol{q}_{\mathfrak{t}_1} \cap \boldsymbol{q}_{\mathfrak{t}_2} = \emptyset$ or $\boldsymbol{q}_{\mathfrak{t}_1} \subseteq \boldsymbol{q}_{\mathfrak{t}_2}$ or $\boldsymbol{q}_{\mathfrak{t}_2} \subseteq \boldsymbol{q}_{\mathfrak{t}_1}$, for each pair $\underbrace{\mathfrak{t}_1 \text{ and } \mathfrak{t}_2}_{\text{compatible}}$, then

$$q'_{\mathsf{tw}}(\vec{x}) = \bigwedge_{S(\vec{z}) \in q} \Big(S(\vec{z}) \vee \bigvee_{\mathfrak{t}\colon S(\vec{z}) \in q_{\mathfrak{t}}} \mathsf{tw}_{\mathfrak{t}} \Big)$$

is a **rewriting** (over H-complete ABoxes)

EL: add (recursive) datalog rules that 'complete' ABox

$\Longrightarrow$ **polynomial datalog rewriting**

(tree witnesses in $\mathcal{EL}$ are compatible)

QL: replace $S(\vec{z})$ with $\displaystyle\bigvee_{\mathcal{T} \models S' \sqsubseteq S} S'(\vec{z})$

$\Longrightarrow$ **polynomial positive existential rewriting**

provided that the number of tree witnesses is **polynomial** and they are **compatible**

not the case in general!

Part 4

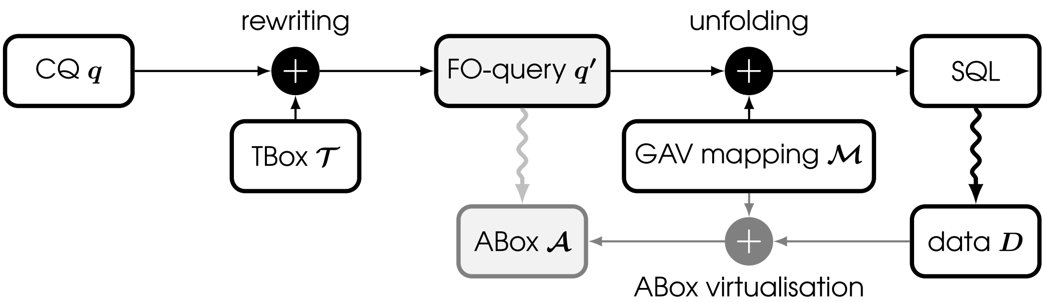
Practical OBDA with Ontop

OBDA system Ontop

http://ontop.inf.unibz.it

- implemented at the Free University of Bozen-Bolzano
  (Mariano Rodríguez-Muro, Martin Rezk, Guohui Xiao)
- open-source
- available as a plugin for Protégé 4, SPARQL end-point,
  OWLAPI and Sesame libraries

OBDA with Databases



Why SQL rewritings are large:

very few for real-world CQs/ontologies

(1) a large number of tree witnesses

(2) large concept/role hierarchies in $\mathcal{T}$

many inclusions in $\mathcal{T}$ follow from Σ and $\mathcal{M}$

(3) multiple definitions of the ontology terms in $\mathcal{M}$

Ontop Example

IMDb (simplified): http://www.imdb.com/interfaces

- database

title

movie ID	title	production year
728	'Django Unchained'	2012
...	...	...

castinfo

person ID	movie ID	person role
n37	728	1
n38	728	1
...	...	...

- dependencies

$$\forall m\ (\exists p, r\ \mathit{castinfo}(p, m, r) \rightarrow \exists t, y\ \mathit{title}(m, t, y)) \qquad \text{(FK)}$$
$$\forall m \forall t_1 \forall t_2\ (\exists y\ \mathit{title}(m, t_1, y) \wedge \exists y\ \mathit{title}(m, t_2, y) \rightarrow (t_1 = t_2)) \qquad (\text{PK}_1)$$
$$\forall m \forall y_1 \forall y_2\ (\exists t\ \mathit{title}(m, t, y_1) \wedge \exists t\ \mathit{title}(m, t, y_2) \rightarrow (y_1 = y_2)) \qquad (\text{PK}_2)$$

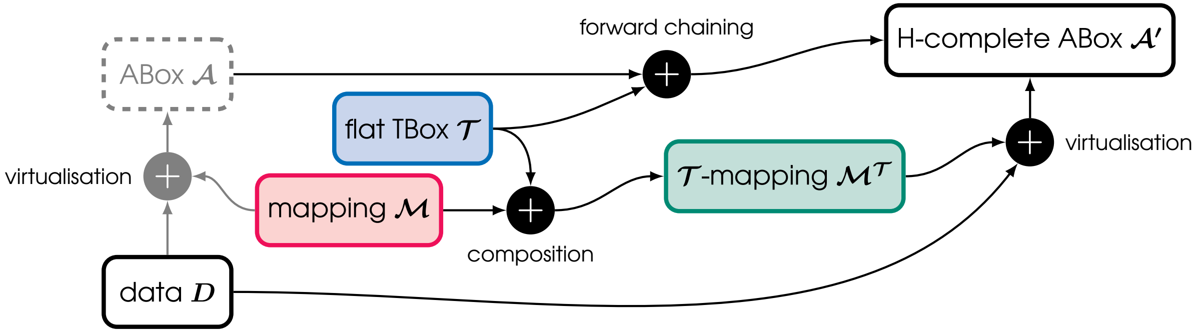
Movie Ontology MO http://www.movieontology.org

$$\mathit{mo{:}Movie} \equiv \exists \mathit{mo{:}title}, \qquad \mathit{mo{:}Movie} \sqsubseteq \exists \mathit{mo{:}year},$$
$$\mathit{mo{:}Movie} \equiv \exists \mathit{mo{:}cast}, \qquad \exists \mathit{mo{:}cast}^- \sqsubseteq \mathit{mo{:}Person}, \ldots$$

Mapping (created by the Ontop development team)

$$\mathit{mo{:}Movie}(m), \mathit{mo{:}title}(m, t), \mathit{mo{:}year}(m, y) \leftarrow \mathit{title}(m, t, y) \qquad (\text{M}_1)$$
$$\mathit{mo{:}cast}(m, p), \mathit{mo{:}Person}(p) \leftarrow \mathit{castinfo}(p, m, r) \qquad (\text{M}_2)$$

Ontop: T-mappings



$\mathcal{T}$	$mo{:}Movie \equiv \exists mo{:}title$, $\quad mo{:}Movie \sqsubseteq \exists mo{:}year$, $mo{:}Movie \equiv \exists mo{:}cast$, $\quad \exists mo{:}cast^- \sqsubseteq mo{:}Person$
$\mathcal{M}$	$mo{:}Movie(\boldsymbol{m}), mo{:}title(\boldsymbol{m}, \boldsymbol{t}), mo{:}year(\boldsymbol{m}, \boldsymbol{y}) \leftarrow title(\boldsymbol{m}, \boldsymbol{t}, \boldsymbol{y})$ (M$_1$) $mo{:}cast(\boldsymbol{m}, \boldsymbol{p}), mo{:}Person(\boldsymbol{p}) \leftarrow castinfo(\boldsymbol{p}, \boldsymbol{m}, \boldsymbol{r})$ (M$_2$)
$\mathcal{M}^{\mathcal{T}}$	$mo{:}Movie(\boldsymbol{m}) \leftarrow title(\boldsymbol{m}, \boldsymbol{t}, \boldsymbol{y})$ by (M$_1$) $\cancel{mo{:}Movie(\boldsymbol{m}) \leftarrow castinfo(\boldsymbol{p}, \boldsymbol{m}, \boldsymbol{r})}$ by (M$_2$) + $\exists mo{:}cast \sqsubseteq mo{:}Movie$

redundant by (FK)

$\forall m\ (\exists p, r\ \textbf{castinfo}(p, m, r) \rightarrow \exists t, y\ \textbf{title}(m, t, y))$

Optimising T-mappings

- using foreign keys (inclusion dependencies)

- using disjunction

$\mathcal{T}$

$$\textit{mo:Actor} \sqsubseteq \textit{mo:Artist}, \qquad \textit{mo:Artist} \sqsubseteq \textit{mo:Person},$$
$$\textit{mo:Director} \sqsubseteq \textit{mo:Person}, \qquad \textit{mo:Editor} \sqsubseteq \textit{mo:Person}, \dots$$

$\mathcal{M}$

$$\textit{mo:Actor}(\boldsymbol{p}) \leftarrow \textit{castinfo}(\boldsymbol{p}, \boldsymbol{m}, \boldsymbol{r}), (\boldsymbol{r} = \mathbf{1}) \qquad (\mathrm{M}_1)$$
$$\dots$$
$$\textit{mo:Editor}(\boldsymbol{p}) \leftarrow \textit{castinfo}(\boldsymbol{p}, \boldsymbol{m}, \boldsymbol{r}), (\boldsymbol{r} = \mathbf{6}) \qquad (\mathrm{M}_6)$$

$\mathcal{M}^{\mathcal{T}}$

$$\textit{mo:Person}(\boldsymbol{p}) \leftarrow \textit{castinfo}(\boldsymbol{p}, \boldsymbol{m}, \boldsymbol{r}), ((\boldsymbol{r} = \mathbf{1}) \lor \cdots \lor (\boldsymbol{r} = \mathbf{6}))$$

Unfolding with Semantic Query Optimisation

Query

$$q(t, y) \leftarrow \textit{mo:Movie}(m), \textit{mo:title}(m, t), \textit{mo:year}(m, y), (y > 2010)$$

Rewriting

$$q'(t, y) \leftarrow \textit{mo:Movie}(m), \textit{mo:title}(m, t), \textit{mo:year}(m, y), (y > 2010)$$

$\mathcal{M}$

$\textit{mo:Movie}(m) \leftarrow \textit{title}(m, t, y)$	(M_1)
$\textit{mo:title}(m, t) \leftarrow \textit{title}(m, t, y)$	(M_2)
$\textit{mo:year}(m, y) \leftarrow \textit{title}(m, t, y)$	(M_3)

Unfolding

$$q^*(t, y) \leftarrow \textit{title}(m, t_0, y_0), \textit{title}(m, t, y_1), \textit{title}(m, t_2, y), (y > 2010)$$

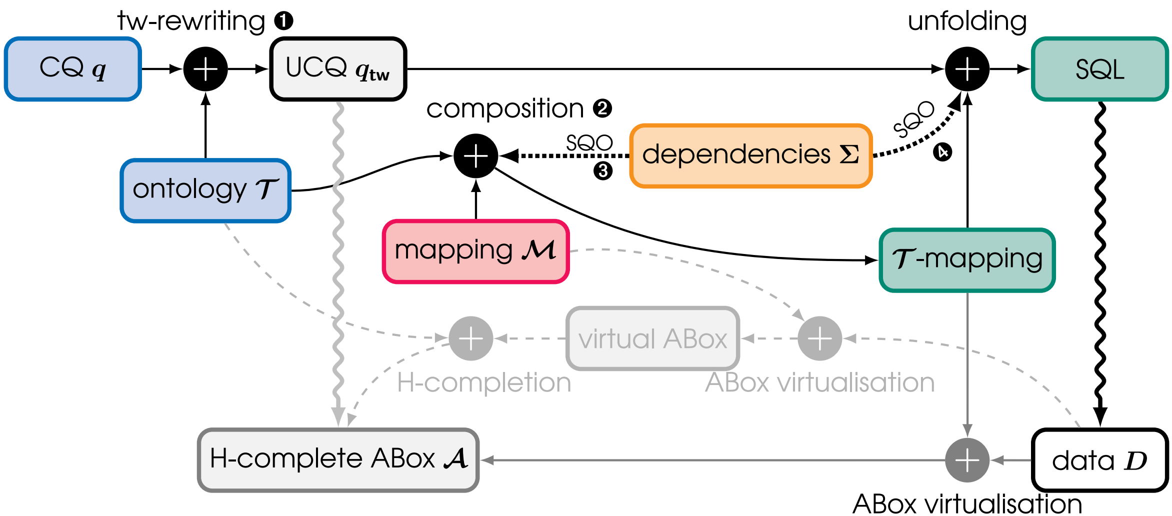
primary keys

$\forall m \forall t_1 \forall t_2 \left(\exists y\, \textit{title}(m, t_1, y) \wedge \exists y\, \textit{title}(m, t_2, y) \rightarrow (t_1 = t_2)\right)$	(PK_1)
$\forall m \forall y_1 \forall y_2 \left(\exists t\, \textit{title}(m, t, y_1) \wedge \exists t\, \textit{title}(m, t, y_2) \rightarrow (y_1 = y_2)\right)$	(PK_2)

Semantic Query Optimisation

$$q^\dagger(t, y) \leftarrow \textit{title}(m, t, y), (y > 2010)$$

Practical OBDA with Ontop



❶ tree-witness rewriting q_{tw} over H-complete ABoxes (no concept/role hierarchies)

❷ $\mathcal{T}$-mapping $=$ system mapping $\mathcal{M} + \mathcal{T}$ makes virtual ABoxes H-complete

❸ $\mathcal{T}$-mapping is simplified using SQO and SQL features

constructed and optimised for $\mathcal{T}$ and Σ only once

❹ unfolding uses SQO to produce small and efficient SQL queries

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